

SADLER UNIT 4 MATHEMATICS METHODS

WORKED SOLUTIONS

Chapter 1 Logarithmic functions

Exercise 1A

Question 1

$$2^3 = 8$$

Question 2

$$7^2 = 49$$

Question 3

$$49^{\frac{1}{2}} = 7$$

Question 4

$$10^3 = 1000$$

Question 5

$$5^4 = 625$$

Question 6

$$4^{2.5} = 32$$

Question 7

$$5^{-2} = 0.04$$

Question 8

$$3^{-2} = \frac{1}{9}$$

Question 9

$$a^y = x$$

Question 10

$$b^c = y$$

Question 11

$$x^p = a$$

Question 12

$$a^3 = x$$

Question 13

$$3^y = 5$$

Question 14

$$2^x = 3$$

Question 15

$$x^4 = 5$$

Question 16

$$3^p = 5$$

Question 17

$$\log_2 64 = 6$$

Question 18

$$\log_3 81 = 4$$

Question 19

$$\log_9 81 = 2$$

Question 20

$$\log_9 27 = \frac{3}{2}$$

Question 21

$$\log_2 0.5 = -1$$

Question 22

$$\log_2 0.25 = -2$$

Question 23

$$\log 100 = 2$$

Question 24

$$\log 0.01 = -2$$

Question 25

$$\log_p r = q$$

Question 26

$$\log_r q = p$$

Question 27

$$\log_2 y = x$$

Question 28

$$\log_3 z = y$$

Question 29

$$\log_5 4 = k$$

Question 30

$$\log_7 3 = y$$

Question 31

$$\log_3 7 = p$$

Question 32

$$\log_e x = y$$

Question 33

$$8^2 = 64$$

$$\therefore 2$$

Question 34

$$2^7 = 128$$

$$\therefore 7$$

Question 35

$$10^4 = 10\,000$$

$$\therefore 4$$

Question 36

$$3^5 = 243$$

$$\therefore 5$$

Question 37

$$\left(\frac{1}{2}\right)^{-1} = 2$$

$$\therefore -1$$

Question 38

$$2^{-4} = \left(\frac{1}{16}\right)$$

$$\therefore -4$$

Question 39

$$6^{-3} = \left(\frac{1}{216}\right)$$

$$\therefore -3$$

Question 40

$$2^{-3} = \left(\frac{1}{8}\right)$$

$$\therefore -3$$

Question 41

$$3^5 = 9^{2.5} = 243$$

$$\therefore 2.5$$

Question 42

$$10^{-3} = 0.001$$

$$\therefore -3$$

Question 43

$$1$$

Question 44

$$7^0 = 1$$

$$\therefore 0$$

Question 45

$$a^0 = 1$$

$$\therefore 0$$

Question 46

$$4^x = 32$$

$$2^{2x} = 2^5$$

$$x = 2.5$$

Question 47

$$1$$

Question 48

$$3$$

Question 49

$$0.699$$

Question 50

$$1.398$$

Question 51

$$0.845$$

Question 52

1.6902

Question 53

1.301

Question 54

1

Question 55

1.322

Question 56

1

Question 57

$$10^c = b$$

- a** c can be negative.
- b** All powers of 10 are positive so $b > 0$.

Exercise 1B

Question 1

$$\log(xz)$$

Question 2

$$\log x^2 + \log y = \log(x^2y)$$

Question 3

$$\log x^2 + \log y^3 = \log(x^2y^3)$$

Question 4

$$\log x^2 - \log y = \log\left(\frac{x^2}{y}\right)$$

Question 5

$$\log\left(\frac{ab}{c}\right)$$

Question 6

$$\log a^3 + \log b^4 - \log c^2 = \log\left(\frac{a^3b^4}{c^2}\right)$$

Question 7

$$\log c^5 - \log c^3 + \log a = \log(c^2a)$$

Question 8

$$\begin{aligned} 2 + \log x &= \log 100 + \log x \\ &= \log(100x) \end{aligned}$$

Question 9

$$\log 1000 - (\log xy) = \log \left(\frac{1000}{xy} \right)$$

Question 10

$$\log 1000 - \log x + \log y = \log \left(\frac{1000y}{x} \right)$$

Question 11

$$\begin{aligned} \log_2 \left(\frac{24}{3} \right) &= \log_2 8 \\ &= 3 \end{aligned}$$

Question 12

$$\begin{aligned} \log_2 \frac{20 \times 8}{10} &= \log_2 16 \\ &= 4 \end{aligned}$$

Question 13

$$\begin{aligned} \log \frac{10^4}{10} &= \log 1000 \\ &= 3 \end{aligned}$$

Question 14

$$3 + 2 - 4 = 1$$

Question 15

$$\begin{aligned} \log_3 \frac{45 \times 2^2}{20} &= \log_3 9 \\ &= 2 \end{aligned}$$

Question 16

$$\begin{aligned}\log_3 4 - \log_3 6^2 - \log_3 9 &= \log_3 \left(\frac{4}{36 \times 9} \right) \\ &= \log_3 \left(\frac{1}{81} \right) \\ &= -4\end{aligned}$$

Question 17

$$\begin{aligned}\log \frac{5}{50} &= \log 0.1 \\ &= -1\end{aligned}$$

Question 18

$$\begin{aligned}\log_a b + \log_a (ab)^2 - \log_a b^3 &= \log_a \left(\frac{b \times a^2 b^2}{b^3} \right) \\ &= 2\end{aligned}$$

Question 19

$$\frac{\log_a b^3}{2 \log_a b} = \frac{3 \log_a b}{2 \log_a b} = 1.5$$

Question 20

$$\begin{aligned}\frac{(\log_a (2^4 \times 3) - \log_a 3)}{\log_a 2} &= \frac{\log 2^4 + \log_a 3 - \log_a 3}{\log_a 2} \\ &= \frac{4 \log_a 2}{\log_a 2} \\ &= 4\end{aligned}$$

Question 21

a $\log_a 6 = \log_a (2 \times 3)$
 $= \log_a 2 + \log_a 3$
 $= p + q$

b $\log_a 18 = \log_a (3^2 \times 2)$
 $= \log_a 3^2 + \log_a 2$
 $= 2\log_a 3 + \log_a 2$
 $= p + 2q$

c $\log_a 12 = \log_a (2^2 \times 3)$
 $= \log_a 2^2 + \log_a 3$
 $= 2\log_a 2 + \log_a 3$
 $= 2p + q$

d $\log_a \frac{2}{3} = \log_a 2 - \log_a 3$
 $= p - q$

e $\log_a (9a^4) = \log_a 9 + \log_a a^4$
 $= \log_a 3^2 + \log_a a^4$
 $= 2\log_a 3 + 4\log_a a$
 $= 2q + 4$

f $\log_a \left(\frac{2}{9}\right) = \log_a 2 - 2\log_a 3$
 $= p - 2q$

Question 22

a $\log_5 49 = \log_5 7^2$
 $= 2\log_5 7$
 $= 2a$

b $\log_5 28 = \log_5 (2^2 \times 7)$
 $= \log_5 2^2 + \log_5 7$
 $= 2\log_5 2 + \log_5 7$
 $= a + 2b$

c $\log_5 1.75 = \log_5 \left(\frac{7}{4}\right)$
 $= \log_5 7 - 2\log_5 2$
 $= a - 2b$

d $\log_5 50 = \log_5 (5^2 \times 2)$
 $= \log_5 5^2 + \log_5 2$
 $= 2\log_5 5 + \log_5 2$
 $= b + 2$

e $\log_5 490 = \log_5 (2 \times 5 \times 7^2)$
 $= \log_5 2 + \log_5 5 + 2\log_5 7$
 $= b + 1 + 2a$
 $= 2a + b + 1$

f $\log_5 700 = \log_5 (7 \times 5^2 \times 2^2)$
 $= \log_5 7 + 2\log_5 5 + 2\log_5 2$
 $= a + 2b + 2$

Question 23

$$\log_a y = x$$
$$y = a^x$$

Question 24

$$\log_a y = \log_a 2x$$
$$y = 2x$$

Question 25

$$\log_a y = \log_a x^3$$
$$y = x^3$$

Question 26

$$\log_a y^2 = \log_a x^3$$
$$y^2 = x^3$$
$$y = x^{\frac{3}{2}}$$

Question 27

$$\log y = \log ax$$
$$y = ax$$

Question 28

$$\log_a y = \log_a a^2 + \log x$$
$$y = a^2 x$$

Question 29

$$\log_a y = \log_a x^{-1}$$
$$y = x^{-1}$$
$$y = \frac{1}{x}$$

Question 30

$$\log_a yx = \log_a a^2$$
$$xy = a^2$$
$$y = \frac{a^2}{x}$$

Question 31

a Initial, $t = 0$.

$$\begin{aligned} S &= 75 - 35 \log 1 \\ &= 75 - 35 \times 0 \\ &= 75 \end{aligned}$$

b When $t = 2$,

$$\begin{aligned} S &= 75 - 35 \log(3) \\ &= 58 \end{aligned}$$

c When $t = 4$

$$\begin{aligned} S &= 75 - 35 \log(5) \\ &= 51 \end{aligned}$$

d $40 = 75 - 35(\log(t + 1))$

$$35 \log(t + 1) = 35$$

$$\log_{10}(t + 1) = 1$$

$$10 = t + 1$$

$$t = 9$$

Question 32

$$R = \log \frac{I}{I_0}$$

a
$$R = \log \frac{1000I_0}{I_0}$$
$$= \log 1000$$
$$= 3$$

b
$$5.4 = \log_{10} \frac{I}{I_0}$$
$$10^{5.4} = \frac{I}{I_0}$$
$$I = 10^{5.4} I_0$$

c
$$I_1 = 10^5 I_0$$
$$I_2 = 10^6 I_0$$
$$\therefore 10 \text{ times}$$

d
$$I_{7.7} = 10^{7.7} I_0$$
$$I_{5.9} = 10^6 I_0$$
$$\frac{10^{7.7} I_0}{10^{5.9} I_0} = 10^{1.8} (\approx 63)$$

Question 33

a Grapes pH = $-\log_{10} 0.0001$
 $= -(-4)$
 $= 4$

b Beer : $-\log_{10}(0.000\ 0316) = x$
 $x = -(-4.5)$
 $= 4.5$

c Urine : $-\log_{10}(0.000\ 000\ 25)$
 $= -(-6.6)$
 $= 6.6$

d Eggs : $-\log_{10}(0.000\ 000\ 016)$
 $= 7.8$

e Blood : $-\log_{10}(0.000\ 000\ 042)$
 $= 7.4$

$$5.25 = -\log_{10}(x)$$

$$5.25 = \log_{10} \frac{1}{x}$$

$$10^{5.25} = \frac{1}{x}$$

$$x = 0.000\ 005\ 623\ 41$$

$\therefore 0.000\ 0056$ moles/litre

Question 34

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right)$$

a $40 = 10 \log_{10} \left(\frac{I}{I_0} \right)$

$$4 = \log_{10} \frac{I}{I_0}$$

$$10^4 = \frac{I}{I_0}$$

$$I = 10^4 I_0$$

b $10^7 I_0$

c $\frac{10^9 I_0}{10^2 I_0} = 10^7$

Exercise 1C

Question 1

$$3^x = 7$$
$$x \log 3 = \log 7$$
$$x = \frac{\log 7}{\log 3}$$

Question 2

$$7^x = 1000$$
$$x \log 7 = \log 1000$$
$$x = \frac{3}{\log 7}$$

Question 3

$$10^x = 27$$
$$x \log 10 = \log 27$$
$$x = \log 27$$
$$= 3 \log 3$$

Question 4

$$2^x = 11$$
$$x \log 2 = \log 11$$
$$x = \frac{\log 11}{\log 2}$$

Question 5

$$3^x = 17$$
$$x \log 3 = \log 17$$
$$x = \frac{\log 17}{\log 3}$$

Question 6

$$\begin{aligned}7^x &= 80 \\x \log 7 &= \log(8 \times 10) \\&= \log 8 + \log 10 \\x &= \frac{\log 8 + 1}{\log 7} \\&= \frac{3 \log 2 + 1}{\log 7}\end{aligned}$$

Question 7

$$\begin{aligned}5^x &= 21 \\x \log 5 &= \log 21 \\x &= \frac{\log 21}{\log 5}\end{aligned}$$

Question 8

$$\begin{aligned}10^x &= 15 \\x \log 10 &= \log 15 \\x &= \log 15\end{aligned}$$

Question 9

$$\begin{aligned}2^x &= 70 \\x \log 2 &= \log 7 + \log 10 \\x &= \frac{1 + \log 7}{\log 2}\end{aligned}$$

Question 10

$$\begin{aligned}(x+2) \log 6 &= \log 17 \\x+2 &= \frac{\log 17}{\log 6} \\x &= \frac{\log 17}{\log 6} - 2\end{aligned}$$

Question 11

$$\begin{aligned}(x+1)\log 3 &= \log(17 \times 3) \\ x+1 &= \frac{\log 17 + \log 3}{\log 3} \\ x &= \frac{\log 17}{\log 3} + \frac{\log 3}{\log 3} - 1 \\ &= \frac{\log 17}{\log 3}\end{aligned}$$

Question 12

$$\begin{aligned}(x-1)\log 8 &= \log 7 \\ x &= \frac{\log 7}{\log 8} + 1 \\ &= \frac{\log 7}{3\log 2} + 1\end{aligned}$$

Question 13

$$\begin{aligned}(x-1)\log 5 &= 2x\log 3 \\ x\log 5 - \log 5 &= 2x\log 3 \\ x\log 5 - 2x\log 3 &= \log 5 \\ x(\log 5 - 2\log 3) &= \log 5 \\ x &= \frac{\log 5}{\log 5 - 2\log 3} \\ &= \frac{\log 5}{\log\left(\frac{5}{9}\right)}\end{aligned}$$

Question 14

$$\begin{aligned}(x+1)\log 2 &= x\log 3 \\ \log 2 &= x\log 3 - x\log 2 \\ x &= \frac{\log 2}{\log 3 - \log 2} \\ &= \frac{\log 2}{\log\left(\frac{3}{2}\right)}\end{aligned}$$

Question 15

$$\begin{aligned}3x \log 4 &= (x+2) \log 5 \\3x \log 4 - x \log 5 &= 2 \log 5 \\x(3 \log 4 - \log 5) &= 2 \log 5 \\x &= \frac{2 \log 5}{3 \log 4 - \log 5} \\&= \frac{2 \log 5}{\log \left(\frac{64}{5} \right)}\end{aligned}$$

Question 16

$$\begin{aligned}(2x+1) \log 3 &= (3x-1) \log 2 \\2x \log 3 + \log 3 &= 3x \log 2 - \log 2 \\2x \log 3 - 3x \log 2 &= -\log 2 - \log 3 \\x(2 \log 3 - 3 \log 2) &= -(\log 2 + \log 3) \\x &= -\frac{(\log 2 + \log 3)}{2 \log 3 - 3 \log 2} \\&= -\frac{\log 6}{\log \left(\frac{9}{8} \right)} \\&= \frac{\log 6}{\log \left(\frac{9}{8} \right)^{-1}} \\&= \frac{\log 6}{\log \left(\frac{8}{9} \right)}\end{aligned}$$

Question 17

$$\begin{aligned}5(2^x) &= 3 - 2^x \times 4 \\9(2^x) &= 3 \\2^x &= \frac{1}{3} \\x \log 2 &= \log \frac{1}{3} \\x &= \frac{\log 3^{-1}}{\log 2} \\&= -\frac{\log 3}{\log 2}\end{aligned}$$

Question 18

$$5^x + 4(5^x \times 5) = 63$$

$$5^x + 20(5^x) = 63$$

$$21(5^x) = 63$$

$$5^x = 3$$

$$x = \frac{\log 3}{\log 5}$$

Question 19

$$(2^x)^2 + 3(2^x) - 18 = 0$$

$$y^2 + 3y - 18 = 0$$

$$(y + 6)(y - 3) = 0$$

$$\therefore y = 6 \quad \text{or} \quad y = -3$$

$$2^x = -6 \quad 2^x = +3$$

$$\text{No such } x \quad x = \frac{\log 3}{\log 2}$$

Question 20

$$(2^x)^2 - 8 \times 2^x + 15 = 0$$

$$y^2 - 8y + 15 = 0$$

$$(y - 5)(y - 3) = 0$$

$$\therefore y = 5 \quad \text{or} \quad y = 3$$

$$2^x = 5 \quad 2^x = 3$$

$$x = \frac{\log 5}{\log 2} \quad \text{or} \quad x = \frac{\log 3}{\log 2}$$

Question 21

$$x = \log_2 7$$

$$2^x = 7$$

$$\log 2^x = \log 7$$

$$x \log 2 = \log 7$$

$$x = \frac{\log 7}{\log 2}$$

Question 22

a
$$\log_3 5 = \frac{\log 5}{\log 3}$$

b
$$\begin{aligned} \log_2 12 &= \frac{\log 12}{\log 2} \\ &= \frac{2\log 2 + \log 3}{\log 2} \\ &= 2 + \frac{\log 3}{\log 2} \end{aligned}$$

c
$$\log_9 15 = \frac{\log 15}{\log 9}$$

d
$$\begin{aligned} \log_9 4 &= \frac{\log 4}{\log 9} \\ &= \frac{2\log 2}{2\log 3} \\ &= \frac{\log 2}{\log 3} \end{aligned}$$

e
$$\log_{2.5} 6.8 = \frac{\log 6.8}{\log 2.5}$$

f
$$\log_{5.4} 9 = \frac{\log 9}{\log 5.4}$$

Question 23

$$T = T_0(0.92)^n$$

$$0.2T_0 = T_0(0.92)^n$$

$$0.2 = 0.92^n$$

$$\log 0.2 = n \log 0.92$$

$$n = \frac{\log 0.2}{\log 0.92}$$

$$= 19.3$$

∴ Needs to pass 20 times.

Question 24

a When $t = 3$,

$$N = 200(2.7)^{0.3}$$

$$= 269.4$$

$$\approx 270$$

b When $t = 5$,

$$N = 200(2.7)^{0.5}$$

$$= 328.6$$

$$\approx 329$$

c $1000 = 200(2.7)^{0.1t}$

$$5 = 2.7^{0.1t}$$

$$\log 5 = 0.1t \log 2.7$$

$$t = (\log_{2.7} 5) \div 0.1$$

$$= 16.2$$

∴ During day 17.

Question 25

$$R = 2.8^{20a}$$

$$51 = 2.8^{20a}$$

$$\log 51 = 20a \log 2.8$$

$$20a = \frac{\log 51}{\log 2.8}$$

$$a = 0.191$$

Question 26

a When $t = 4$,

$$\begin{aligned} N &= 100\,000 + 150\,000(1.1)^{-0.8(4)} \\ &= 210\,569 \\ &\approx 211\,000 \end{aligned}$$

b When $t = 8$

$$\begin{aligned} N &= 100\,000 + 150\,000(1.1)^{-0.8(8)} \\ &= 181\,503 \\ &\approx 182\,000 \end{aligned}$$

c $135\,000 = 100\,000 + 150\,000(1.1)^{-0.8t}$

$$\begin{aligned} \frac{35}{150} &= (1.1)^{-0.8t} \\ \log\left(\frac{7}{30}\right) &= -0.8t \log 1.1 \end{aligned}$$

$$\begin{aligned} t &= \left[\frac{\left(\log \frac{7}{30}\right)}{\log 1.1} \right] \div -0.8 \\ t &= 19.086 \end{aligned}$$

Approximately 19 weeks after the campaign ceases.

Question 27

$$P = 10\,000(1.08)^x$$

a When $x = 3$, $P = 10\,000(1.08)^3$
 $= \$12597.12$

b When $x = 7$, $P = 10\,000(1.08)^7$
 $= \$17138.24$

c $50\,000 = 10\,000(1.08)^x$
 $5 = 1.08^x$
 $\log 5 = x \log 1.08$
 $x = \frac{\log 5}{\log 1.08}$
 $= 20.9$

\therefore 21 years

d i $50\,000 = 10\,000(1.1)^x$
 $5 = 1.1^x$
 $x = 16.89$
 $\therefore \sim 17$ years

ii $10\,000 \times 1.14^8 = 28\,525.86$

$$50\,000 = 28\,525.86 \times 1.1^x$$
$$\log \left(\frac{50\,000}{28\,525.86} \right) = x \log 1.1$$
$$x = 5.89$$

$\therefore \sim 6$ years

\therefore 14 years in total.

e

$$20\,000 = 10\,000 \times R^5$$

$$2 = R^5$$

$$\log 2 = 5 \log R$$

$$\log R = \frac{\log 2}{5}$$

$$\log R = 0.0602$$

$$\therefore R = 1.1487$$

\therefore 14.9%

Exercise 1D

Question 1

$$\log_e e = 1$$

Question 2

$$\begin{aligned}\log_e \left(\frac{1}{e}\right) &= \log_e e^{-1} \\ &= -1 \log_e e \\ &= -1\end{aligned}$$

Question 3

$$\begin{aligned}\log_e e^3 &= 3 \log_e e \\ &= 3\end{aligned}$$

Question 4

$$\begin{aligned}\log_e \sqrt{e} &= \log_e e^{\frac{1}{2}} \\ &= \frac{1}{2} \log_e e \\ &= \frac{1}{2}\end{aligned}$$

Question 5

$$\begin{aligned}\ln e^{\frac{1}{3}} &= \frac{1}{3} \log_e e \\ &= \frac{1}{3}\end{aligned}$$

Question 6

$$\begin{aligned}\ln e^{-\frac{1}{2}} &= -\frac{1}{2} \log_e e \\ &= -\frac{1}{2}\end{aligned}$$

Question 7

$$\begin{aligned}\ln e^{-3} &= -3 \log_e e \\ &= -3\end{aligned}$$

Question 8

$$\begin{aligned}\ln e^{-\frac{1}{3}} &= -\frac{1}{3} \log_e e \\ &= -\frac{1}{3}\end{aligned}$$

Question 9

$$\begin{aligned}e^{x+1} &= 7 \\ \ln e^{x+1} &= \ln 7 \\ (x+1) \ln e &= \ln 7 \\ x &= \ln 7 - 1\end{aligned}$$

Question 10

$$\begin{aligned}e^{x+3} &= 50 \\ \ln e^{x+3} &= \ln 50 \\ (x+3) \ln e &= \ln 50 \\ x &= \ln 50 - 3\end{aligned}$$

Question 11

$$\begin{aligned}e^{x-3} &= 100 \\ (x-3) \ln e &= \ln 100 \\ x &= \ln 100 + 3 \\ &= 2 \ln 10 + 3\end{aligned}$$

Question 12

$$e^{2x+1} = 15$$

$$(2x+1)\ln e = \ln 15$$

$$2x = \ln 15 - 1$$

$$x = \frac{\ln 15 - 1}{2}$$

Question 13

$$5e^{3x-1} = 3000$$

$$e^{3x-1} = 600$$

$$(3x-1)\ln e = \ln 600$$

$$3x = \ln 600 + 1$$

$$x = \frac{\ln 600 + 1}{3}$$

Question 14

$$4e^{x+2} + 3e^{x+2} = 7000$$

$$7e^{x+2} = 7000$$

$$e^{x+2} = 1000$$

$$(x+2)\ln e = \ln 1000$$

$$x = \ln 1000 - 2$$

$$= 3\ln 10 - 2$$

Question 15

$$y^2 - 30y + 200 = 0$$

$$(y-20)(y-10) = 0$$

$$y = 10 \quad \text{or} \quad y = 20$$

$$e^x = 10 \quad e^x = 20$$

$$x \ln e = \ln 10 \quad x \ln e = \ln 20$$

$$x = \ln 10 \quad x = \ln 20$$

Question 16

$$\log_7 2 = \frac{\ln 2}{\ln 7}$$

Question 17

$$\begin{aligned}\log_2 21 &= \frac{\ln 21}{\ln 2} \\ &= \frac{\ln 3 + \ln 7}{\ln 2}\end{aligned}$$

Question 18

$$\begin{aligned}\log_3(2^3 \times 5^2) &= \frac{\ln 2^3 + \ln 5^2}{\ln 3} \\ &= \frac{3\ln 2 + 2\ln 5}{\ln 3}\end{aligned}$$

Question 19

$$\begin{aligned}\log_5(5^2 \times 2) &= \frac{\ln 5^2 + \ln 2}{\ln 5} \\ &= \frac{2\ln 5 + \ln 2}{\ln 5} \\ &= 2 + \frac{\ln 2}{\ln 5}\end{aligned}$$

Question 20

$$\begin{aligned}\log_6 9 &= \frac{\ln 3^2}{\ln 2 + \ln 3} \\ &= \frac{2\ln 3}{\ln 2 + \ln 3}\end{aligned}$$

Question 21

$$\log_9 6 = \frac{\ln 2 + \ln 3}{2\ln 3}$$

Question 22

$$\begin{aligned}\log_4(2^2 \times 3 \times 5)^2 &= \frac{2\ln 2 + \ln 3 + 2\ln 5}{2\ln 2} \\ &= 1 + \frac{\ln 3 + 2\ln 5}{2\ln 2}\end{aligned}$$

Question 23

$$\log_8(2^2 \times 5 \times 11) = \frac{2\ln 2 + \ln 5 + \ln 11}{3\ln 2}$$

Question 24

$$\begin{aligned}A &= 2000e^{-t} \\ \frac{A}{2000} &= e^{-t} \\ \ln \frac{A}{2000} &= -t \\ t &= -\ln \frac{A}{2000} \\ &= \ln \left(\frac{A}{2000} \right)^{-1} \\ &= \ln \left(\frac{2000}{A} \right)\end{aligned}$$

a When $A = 1500$,

$$\begin{aligned}t &= \ln \left(\frac{2000}{1500} \right) \\ &= 0.288\end{aligned}$$

b When $A = 500$,

$$\begin{aligned}t &= \ln \left(\frac{2000}{500} \right) \\ &= 1.386\end{aligned}$$

c When $A = 50$,

$$\begin{aligned}t &= \ln \left(\frac{2000}{50} \right) \\ &= 3.689\end{aligned}$$

Question 25

a $22\,300\,000e^{0.02t} = 32\,000\,000$

$$e^{0.02t} = \frac{320}{223}$$

$$0.02t = \ln\left(\frac{320}{223}\right)$$

$$t = 18.06$$

$$2010 + 18 \sim 2028$$

b $223e^{0.02t} = 450$

$$e^{0.02t} = \frac{450}{223}$$

$$0.02t = \ln\left(\frac{450}{223}\right)$$

$$t = 35.1$$

$$2010 + 35 \sim 2045$$

Question 26

a $N \approx 5000e^{0.55t}$

$$80\,000 = 5000e^{0.55t}$$

$$16 = e^{0.55t}$$

$$0.55t = \ln 16$$

$$t = 5.04$$

$$\sim 5 \text{ days}$$

b $750\,000 = 5000e^{0.55t}$

$$150 = e^{0.55t}$$

$$0.55t = \ln 150$$

$$t = 9.11$$

$$\sim 9 \text{ days}$$

Exercise 1E

Question 1

a

The graph of $y = \log_2(x+8)$ is $y = \log_2 x$ translated 8 units left.

$$\log_2(x+8) = 0$$

$$2^0 = x+8$$

$$x = 1-8$$

$$= -7$$

$$\therefore (-7, 0)$$

b

$$y\text{-int, } x = 0$$

$$\log_2 8 = 3$$

$$\therefore (0, 3)$$

Question 2

$$y = \log_p x$$

$$p^y = x$$

$$y = 0, x = 1$$

$$\therefore (1, 0)$$

Question 3

$$1 = \log_a x$$

$$a^1 = x$$

$$\therefore (a, 1)$$

Question 4

a $y = \log_p x$

$$p^y = x$$

$$p^y \neq 0.$$

\therefore vertical asymptote at $x = 0$, y-axis

b $\log_p(x-3) = y$ ($y = \log_p x$ translated 3 units right)

$$p^y = x - 3$$

$$p^y + 3 = x$$

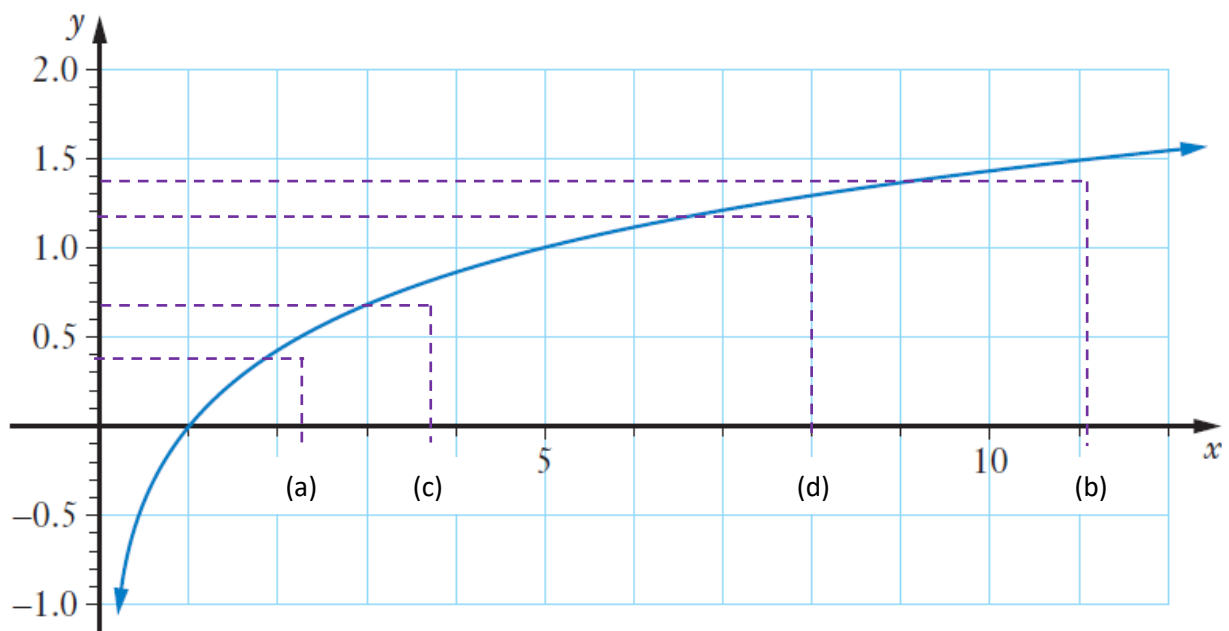
$$p^y \neq 0 \Rightarrow x \neq 3$$

$\therefore x = 3$ is a vertical asymptote

c $y = \log_p x - 3$ ($y = \log_p x$ translated down 3 units)

\therefore Vertical asymptote is $x = 0$

Question 5



a $x \approx 2.2$

b $x \approx 11.2$

c $x - 5^{0.8} = 0$
 $x = 5^{0.8}$
 $\log_5 x = 0.8$
 $x \approx 3.6$

d $\log_5(x-1) = 1.3$
 $\log_5 x = 1.3$
 $x \approx 8.1$

The graph of $y = \log_5(x-1)$ is $y = \log_5 x$ translated 1 unit right

$\therefore \log_5(x-1) = 1.3$

$x \approx 9.1$

$$\begin{aligned} \mathbf{e} \quad \log_5 x &= \frac{1}{2} \\ x &= 5^{\frac{1}{2}} \\ &= 2.236 \end{aligned}$$

$$\begin{aligned} \log_5 x &= 1.5 \\ x &= 5^{1.5} \\ &= 11.180 \end{aligned}$$

$$\begin{aligned} x &= 5^{0.8} \\ &= 3.624 \end{aligned}$$

$$\begin{aligned} \log_5(x-1) &= 1.3 \\ 5^{1.3} &= x-1 \\ x &= 5^{1.3} + 1 \\ &= 9.103 \end{aligned}$$

Question 6

$y = \log_a x$ is the middle graph as it passes through (1, 0)

When $x = 2$, $y = 1$

$$1 = \log_a 2 \therefore a = 2$$

$y = \log_2(x-b)$ has been translated 4 units right.

$$\therefore b = 4$$

$y = \log_2 x + c$ has been vertically translated 3 units up

$$\therefore c = 3$$

Exercise 1F

Question 1

a $N = -\log_{10}(6.4 \times 10^{-8})$
 $= 7.19$

b $9.5 = -\log_{10}(2L)$
 $= \log_{10}(2L)^{-1}$

$$10^{9.5} = \frac{1}{2L}$$

$$2L = \frac{1}{10^{9.5}}$$

$$L = 1.58 \times 10^{-10}$$

Question 2

a $x = \frac{1}{\log 2} \times \log \frac{f_2}{f_1}$
 $= \frac{1}{\log 2} \times \log \left(\frac{50}{20} \right)$
 $= 1.32$ octaves

b $3 = \frac{1}{\log 2} \times \log \left(\frac{f_2}{f_1} \right)$

$$3 \log 2 = \log \left(\frac{f_2}{f_1} \right)$$

$$\log 2^3 = \log \left(\frac{f_2}{f_1} \right)$$

$$8 = \frac{f_2}{f_1}$$

$$f_2 = 8f_1$$

Question 3

a $\text{pH} = -\log[\text{H}^+]$

$$7 = -\log[\text{H}^+]$$

$$7 = \log_{10}[\text{H}^+]^{-1}$$

$$10^7 = [\text{H}^+]^{-1}$$

$$= \frac{1}{\text{H}^+}$$

$$\text{H}^+ = \frac{1}{10^7}$$

$$= 10^{-7} \text{ moles/L}$$

b $\text{pH} = -\log[0.01]$

$$= -[-2]$$

$$= 2$$

Question 4

a $\ln\left(\frac{0.2}{0.8}\right) = -1.39$

b $4 = \ln\left(\frac{p}{1-p}\right)$

$$e^4 = \frac{p}{1-p}$$

$$e^4(1-p) = p$$

$$e^4 - e^4p = p$$

$$e^4 = p + pe^4$$

$$= p(1 + e^4)$$

$$p = \frac{e^4}{(1 + e^4)}$$

$$= 0.98$$

$$\mathbf{c} \quad 0 = \ln\left(\frac{p}{1-p}\right)$$

$$e^0 = \frac{p}{1-p}$$

$$1-p = p$$

$$1 = 2p$$

$$p = \frac{1}{2}$$

Negative logit $0 < p < \frac{1}{2}$. Not likely to occur.

$$\mathbf{d} \quad \ln\left(\frac{x}{1-x}\right) = k$$

$$\frac{x}{1-x} = e^k$$

$$x = e^k - e^k \times x$$

$$x + xe^k = e^k$$

$$x(1 + e^k) = e^k$$

$$x = \frac{e^k}{(1 + e^k)}$$

As $k \rightarrow +\infty$, $x \rightarrow 1$

As $k \rightarrow -\infty$, $x \rightarrow 0$

Question 5

See textbook for full answer.

Question 6

See textbook for full answer.

Miscellaneous exercise one

Question 1

$$15x^2$$

Question 2

$$\frac{d}{dx}(x^3 + x) = 3x^2 + 1$$

Question 3

$$\begin{aligned} & \frac{(2x+5) \times 1 - (x-3) \times 2}{(2x+5)^2} \\ &= \frac{2x+5-2x+6}{(2x+5)^2} \\ &= \frac{11}{(2x+5)^2} \end{aligned}$$

Question 4

$$\begin{aligned} & 4(x^3 + 1)^3 \times 3x^2 \\ &= 12x^2(x^3 + 1)^3 \end{aligned}$$

Question 5

$$e^x$$

Question 6

$$2e^x$$

Question 7

$$10e^x$$

Question 8

$$e^x + 6x + 3x^2$$

Question 9

$$5e^{5x}$$

Question 10

$$3 \times 4e^{4x} = 12e^{4x}$$

Question 11

$$3 \times 2e^{2x} = 6e^{2x}$$

Question 12

$$\begin{aligned} 2 \times 3e^{3x} + 3 \times 2e^{2x} \\ = 6e^{3x} + 6e^{2x} \\ = 6e^{2x}(e^x + 1) \end{aligned}$$

Question 13

$$3^4 = 81$$

Question 14

$$6^3 = 216$$

Question 15

$$2^{-2} = 0.25$$

Question 16

$$a^c = b$$

Question 17

$$a^b = c$$

Question 18

$$b^c = a$$

Question 19

$$c^b = a$$

Question 20

$$x^5 = 2$$

Question 21

$$\log_2 8 = 3$$

Question 22

$$\log_5 25 = 2$$

Question 23

$$\log_4 0.25 = -1$$

Question 24

$$\log_2 0.125 = -3$$

Question 25

$$\log_7 y = x$$

Question 26

$$\log_a p = 2$$

Question 27

$$\log_{10} z = y$$

Question 28

$$\log_e x = y$$

Question 29

$$\begin{aligned}\log_2 2^5 &= 5\log_2 2 \\ &= 5\end{aligned}$$

Question 30

$$\begin{aligned}\log_5 5^3 \\ &= 3\log_5 5 \\ &= 3\end{aligned}$$

Question 31

$$\log_{10} 10 = 1$$

Question 32

$$\begin{aligned}\log 10^3 \\ &= 3\log 10 \\ &= 3\end{aligned}$$

Question 33

$$\begin{aligned}5 + \log_e e \\ &= 5 + 1 \\ &= 6\end{aligned}$$

Question 34

$$\begin{aligned} &4 - 2 \ln e \\ &= 4 - 2 \\ &= 2 \end{aligned}$$

Question 35

$$\begin{aligned} &6 \ln e^{0.5} \\ &= 0.5 \times 6 \\ &= 3 \end{aligned}$$

Question 36

$$\begin{aligned} &\log_2 2^3 + \ln(e^{-1}) \\ &= 3 \log_2 2 - \log_e e \\ &= 3 - 1 \\ &= 2 \end{aligned}$$

Question 37

$$\begin{aligned} &\log_a a^0 \\ &= 0 \end{aligned}$$

Question 38

$$\begin{aligned} &\log_a a \\ &= 1 \end{aligned}$$

Question 39

$$\begin{aligned} &3 \log_a a \\ &= 3 \end{aligned}$$

Question 40

$$\begin{aligned} &\log_a a^{0.5} \\ &= 0.5 \end{aligned}$$

Question 41

$$\begin{aligned}\ln e^{x+1} &= \ln 12 \\(x+1)\ln e &= \ln 12 \\x &= \ln 12 - 1\end{aligned}$$

Question 42

$$\begin{aligned}\ln e^{x+2} &= \ln 25 \\(x+2)\ln e &= \ln 25 \\x &= \ln 25 - 2\end{aligned}$$

Question 43

$$\begin{aligned}\ln e^{x-1} &= \ln 150 \\(x-1)\ln e &= \ln 150 \\x &= \ln 150 + 1\end{aligned}$$

Question 44

$$\begin{aligned}\ln e^{2x+1} &= \ln 34 \\(2x+1)\ln e &= \ln 34 \\2x &= \ln 34 - 1 \\x &= \frac{\ln 34 - 1}{2}\end{aligned}$$

Question 45

$$\begin{aligned}5e^{x+1} + 3e^{x+1} &= 200 \\8e^{x+1} &= 200 \\e^{x+1} &= 25 \\(x+1)\ln e &= \ln 25 \\x &= \ln 25 - 1\end{aligned}$$

Question 46

$$e^{2x} - 12e^x = -35$$

$$(e^x)^2 - 12e^x + 35 = 0$$

$$(e^x - 7)(e^x - 5) = 0$$

$$e^x = 7 \quad \text{or} \quad e^x = 5$$

$$\ln e^x = \ln 7 \quad \ln e^x = \ln 5$$

$$x = \ln 7 \quad x = \ln 5$$

Question 47

$$\log x^3 + \log y$$

$$= \log(x^3 y)$$

Question 48

$$\log x^2 - \log y^3$$

$$= \log\left(\frac{x^2}{y^3}\right)$$

Question 49

$$\log a^2 + \log b - \log c^3$$

$$= \log\left(\frac{a^2 b}{c^3}\right)$$

Question 50

$$\log 1000 + \log x$$

$$= \log(1000x)$$

Question 51

$$\ln e^2 + \ln x$$

$$= \ln(xe^2)$$

Question 52

$$\begin{aligned} & \ln e^3 - \ln x + \ln y^2 \\ &= \ln \left(\frac{e^3 y^2}{x} \right) \end{aligned}$$

Question 53

$$\begin{aligned} P &= P_0 e^{0.1t} \\ 5P_0 &= P_0 e^{0.1t} \\ 5 &= e^{0.1t} \\ \ln 5 &= \ln e^{0.1t} \\ &= 0.1t \ln e \\ \ln 5 &= 0.1t \\ t &= \frac{\ln 5}{0.1} \\ &\approx 16.09 \text{ years} \end{aligned}$$

$$2010 + 16 \sim 2026$$

Question 54

$$\begin{aligned}\mathbf{a} \quad v &= \int a \, dt \\ &= \int 0.1e^{0.1t} \, dt \\ &= e^{0.1t} + c\end{aligned}$$

When $t = 0, v = 0$

$$\begin{aligned}0 &= e^{0.1(0)} + c \\ &= 1 + c\end{aligned}$$

$$c = -1$$

$$v = (e^{0.1t} - 1) \text{ m/s}$$

When $t = 10$

$$\begin{aligned}v &= e^{0.1(10)} - 1 \\ &= (e^1 - 1) \text{ m/s} \\ &= 1.72 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad x &= \int v \, dt \\ &= \int (e^{0.1t} - 1) \, dt \\ &= 10e^{0.1t} - t + c\end{aligned}$$

When $t = 0, x = 0$

$$\begin{aligned}0 &= 10e^{0.1(0)} - 0 + c \\ c &= -10\end{aligned}$$

$$x = (10e^{0.1t} - t - 10) \text{ m}$$

When $t = 10$

$$\begin{aligned}x &= 10e^{0.1(10)} - 10 - 10 \\ &= 10e - 20 \\ &= 10(e - 2) \text{ m} \\ &= 7.18 \text{ m}\end{aligned}$$

c

$$\begin{aligned}x(T+1) - x(T) &= 10e^{0.1(T+1)} - (T+1) - 10 - (10e^{0.1T} - T - 10) \\ &= 10e^{0.1T} e^{0.1} - T - 1 - 10 - 10e^{0.1T} + T + 10 \\ &= 10e^{0.1T} e^{0.1} - 10e^{0.1T} - 1 \\ &= 10e^{0.1T} (e^{0.1} - 1) - 1 \text{ m}\end{aligned}$$

d In the third second, $T=2$

$$10e^{0.1(2)}(e^{0.1} - 1) - 1 = 0.285 \text{ m}$$

e In the tenth second, $T=9$

$$10e^{0.1(9)}(e^{0.1} - 1) - 1 = 1.587 \text{ m}$$